



The SAT

**TUTORING**
MACHINES

SAT Manual

by Kimani Williams, PhD

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Note From The Author

Kimani “The Machine” Williams, PhD

Ever since I have known myself, I have been tutoring my friends and fellow students in academia. I have been blessed with a knack for simplifying what seems complicated to students. After high school, I left my home country of Jamaica to go to Macalester College, a small liberal arts college in Minnesota. After four wonderful years of Minnesota in the frigid tundra, it was time to return to a climate more favorable to a Jamaican, so I decided to further my studies by joining the PhD program in Electrical Engineering at the University of California, Santa Barbara. While working on my doctorate, I started tutoring part-time for a large test prep company. It so happens, that I am phenomenal at dramatically improving students’ test scores. Soon, I was tutoring so many students that I began to feel more like a part-time PhD student and a full-time tutor.

After completing my PhD in Electrical Engineering, I decided to fully devote myself to my passion for teaching and tutoring, which led me to start my own company, Tutoring Machines, LLC. In my own microcosmic way, I am changing the world, one student at a time as I help students reach their goals through higher education.

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General Approach to Reading Passages

- **Read the blurb (text that precedes the passage), then speed read the passage.**

The goal is to get the main idea and to map the passage. The passages tend to have a lot of line reference questions that follow, so it is possible to do well on the passages by going to the questions and then going to find the answers to the questions. However, it is recommended that you speed read the passage first.

- **Attack specific questions first, then general questions.**

Specific questions are typically less time-consuming and easier than general questions.

- **Refer back to the passage to find the answer to the question.**

The answers to the questions are **ALWAYS** in the passage.

- **Articulate the answer to the question in your own words.**

This is a crucial step. Doing this makes eliminating wrong answer choices easier.

- **Pick the answer choice that is closest to your answer.**

Eliminate answers that are not mentioned in the passage or contradicted by the passage.

Chronology

The best answer is always supported by the passage. The answers are typically organized in chronological order in the passage.

Questions 1-10 are based on the following passage.

This passage is adapted from a 1911 novel. Here, the main character, a young man who has received a large inheritance, discusses his desire to take up a hobby.

Line
5 Three men sat in the Cosmic Club discussing the question: “What’s the matter with Jones?” Waldemar, the oldest of the conferees, was the operator of an important and decent newspaper. Beside him sat
5 Bertram, the club idler, slender and languidly elegant. The third member of the conference was Jones himself. Average Jones had come by his nickname inevitably. His parents had foredoomed him to it when they furnished him with the initials A.V.R.E. as
10 preface to his birthright of J for Jones.

Now, at twenty-seven, Average Jones looked back over the five years since his graduation from college and wondered why he had not in some manner justified the parting words of his favorite professor.
15 “You have one rare faculty, Jones. You can, when you choose, sharpen the pencil of your mind to a very fine point. Specialize, my boy, specialize.” If the recipient of this admonition had specialized in anything, it was in life. He might have continued in that path
20 indefinitely, but for two influences. One was an irruptive craving within him to take some part in the dynamic activities of the surrounding world. The other was the “freak” inheritance from his late and little-lamented uncle, Adrian Van Reypen Egerton, who had
25 left to Average Jones future expectations of some ten millions.

In a will whose cynical humor was the topic of its day, Mr. Egerton jeered posthumously at the public which he had deprived, and promised compensation
30 through his heir, Average Jones. “Therefore,” he had written, “I give and bequeath to the said Adrian Van Reypen Egerton Jones, the residue of my property, the principal to be taken over by him at such time as he shall have completed five years of continuous
35 residence in New York City. After such time the virus of the metropolis will have worked through his entire being. He will squander his undeserved fortune, thus completing the vicious circle, and returning the millions acquired by my political activities, in a
40 poisoned shower back to the city.”

“And now,” remarked Waldemar in his heavy, rumbling voice, “you aspire to disappoint that good old man.” Average Jones was bent on proving his
45 uncle wrong by not squandering away the fortune he inherited. “Exactly. Anyway, I’ve no taste for

dissipation. I want action; I want something to do.” proclaimed Average Jones. Waldemar laughed. “Why don’t you take up a hobby, Mr. Jones?”

“What kind of a hobby?” responded Average
50 Jones. “Any kind. The club is full of hobby-riders,” responded Waldemar. “Look at old De Gay; he collects venomous insects from all over the world. Fenton, over there, has the finest collection of circus posters and stamps in the world. I’ve often wished I
55 had the time to be a crank. It’s a happy life.”

“What line would you choose?” asked Bertram languidly. Waldemar had always pondered the validity of advertisements in his newspapers trying to decipher the ones that were scams from the genuine ones. “If
60 one could take the time to follow advertisements – sort out the true claims from the codswallop – but it would mean all one’s leisure.” answered Waldemar.

“Would it be so demanding? The ordinary run of advertising is nothing more than an effort to sell something by yelling in print,” said Average Jones,
65 smiling.

“Is it? Well perhaps you don’t look in the right place.” Waldemar reached for the morning’s copy of the Universal and ran his eye down the columns
70 of “classified” matter. “Hark,” he said, and read: “WANTED – A venerable looking man with white beard and medical degree. Good pay to right applicant.” “What’s that?” asked Average Jones with awakened interest. “Only a quack medical
75 doctor looking for a stall to impress their come-ons,” explained Waldemar.

Average Jones leaned over to scan the paper in his turn. “Here’s one,” said he, and read: WANTED – Performer on B-flat trombone. Can use at once.
80 Apply with instrument, after 1 p.m. 300 East 100th Street. “That seems ordinary enough,” said Waldemar. “What’s it doing in a daily paper? There must be – er – technical publications, you know, for this sort of demand.” “When Average’s words come slow, you’ve got him interested,” commented Bertram. “Sure sign.”

“I’d like to take a look at this business,” declared Average Jones with sudden conviction. “It looks to me like something to do.” Average Jones had found his hobby. He then decided to spend his time proving to
90 unsuspecting subjects whether they were about to be defrauded by advertisements posted in newspapers. He proved himself quite adroit at such a task. It turns out that Average Jones was not average after all.

1

Which choice best describes what happens in the passage?

- A) a casual conversation among acquaintances
- B) a meeting to discuss the distribution of an inheritance
- C) a heated confrontation
- D) a chance encounter among old friends

2

The main point of the first paragraph is to

- A) describe the setting, the Cosmic Club, in which the meeting was taking place.
- B) explain the importance of maintaining friendships.
- C) describe the relationship between Average Jones and his parents.
- D) introduce the characters and state the purpose of their gathering.

3

As it is used in line 46, “dissipation” most nearly means

- A) distrust.
- B) being wasteful.
- C) business ventures.
- D) mediocrity.

4

Mr. Egerton’s will intends to make amends to the public of New York by

- A) distributing his millions to the citizens of New York.
- B) sincerely apologizing for his wrongdoings.
- C) creating a foundation for those he harmed.
- D) leaving his property to Average Jones.

5

Which of the following statements most accurately expresses the opinion of Average Jones about the ad requesting the trombone performer?

- A) Average Jones thinks the ad is an excellent work opportunity.
- B) Average Jones is considering responding to the ad.
- C) Average Jones is concerned that the ad is a sham request.
- D) Average Jones believes it is a repeated ad in the daily paper

6

As used in line 61, “codswallop” most nearly means

- A) baseless claims.
- B) classifieds.
- C) comedic claims.
- D) headlines.

7

It can logically be inferred from the passage that the reason Waldemar does not have a hobby is that

- A) he will retire soon.
- B) his life is occupied with other pursuits.
- C) he lives a boring life.
- D) he is consumed with figuring out the problems of Average Jones.

8

Which choice provides the best evidence for the answer to the previous question?

- A) Lines 2-4 (“Waldemar, the . . . newspaper”)
- B) Lines 42-43 (“You aspire . . . man”)
- C) Lines 54-55 (“I’ve often . . . crank”)
- D) Line 81 (“That seems . . . enough”)



Pronoun Forms

Know the difference between words that sound the same but have different meanings (homophones).

- Example:* **Their** - belonging to them (e.g. **Their** house was decorated with the most Christmas lights.)
 There - refers to being at a place / shows that something exists (e.g. I will be **there** soon. **There** is a key under the mat.)
 They're - contraction for "they are" (e.g. **They're** going to the prom together.)
- Its** - belonging to something (e.g. The dog loves to wag **its** tail.)
It's - contraction for "it is" (e.g. **It's** time to go.)
- Whose** - possessive form of who (e.g. **Whose** book is this?)
Who's - contraction for "who is" (e.g. **Who's** watching the TV?)

Mayweather vs. Pacquiao

My friend and **1** me decided to order the Mayweather-Pacquiao fight from **2** him. It was the most anticipated fight in the past quarter century. **3** There was a lot of hype surrounding the fight and many big-name celebrities showed up on May 2nd at the MGM Grand in Las Vegas to watch the fight.

The promoters of the fight only sold 500 tickets to the public. The tickets went on sale and within 60 seconds **4** it were all sold out. The average price for the tickets sold to the public was roughly \$7,000 and ringside seats were going for as much as \$345,000.

Finally, the anticipation was over and fight night was here. As the fight progressed, **5** he showed why he is considered one of the best defensive fighters of all time.

1

- A) NO CHANGE
- B) he
- C) him
- D) I

2

- A) NO CHANGE
- B) the Pay Per View Channel
- C) her
- D) them

3

- A) NO CHANGE
- B) Their
- C) They're
- D) They are

4

- A) NO CHANGE
- B) they
- C) them
- D) that

5

- A) NO CHANGE
- B) him
- C) it
- D) Mayweather



Drill 3: Idioms and More

The company had to use an **1** innovative engineering method in order to increase the battery life of the new smart phone.

His performance in the arts was as impressive **2** than his performance in the sciences.

I would not have survived high school if it wasn't for Mrs. Williams. She was firm **3** around her decisions when it came to discipline.

Of the two scientists, Einstein and Newton, many people continually debate who of the two **4** has been most influential on the world of science.

1

- A) NO CHANGE
- B) innovation engineering
- C) innovate engineering
- D) innovate engineer

2

- A) NO CHANGE
- B) as
- C) if
- D) then

3

- A) NO CHANGE
- B) into
- C) regarding
- D) on

4

- A) NO CHANGE
- B) have been more
- C) has been more
- D) has being more

“Must be” and “Equivalent” Questions

“Must be” questions also have variables in the answer choices. However, for these questions you will typically need to pick multiple numbers to get to the correct answer. Pick numbers until 3 answer choices are eliminated and 1 answer choice is left. “Equivalent” questions are great making up numbers questions since these questions will typically have variables in the answer choices.

7

If x is a positive integer, which of the following expressions must be an odd integer?

- A) x^2
- B) 5^x
- C) $x + 3$
- D) x^3

8

If $\frac{x}{y} = 3$, then the value of $\frac{6y}{x}$ must be

- A) 0
- B) 1
- C) 2
- D) 4

30

If a and b are real numbers such that $a > 2$ and $b < -2$, then which of the following choices must be true?

- A) $\frac{a}{b} > 1$
- B) $a^2 + 2 > b^2 + 2$
- C) $a^{-2} > b^{-2}$
- D) $\frac{a}{4} - 3 > \frac{b}{4} - 3$

12

Which of the following is equivalent to $\frac{2x^2 + 7x}{(2x + 3)}$?

- A) $x + 2$
- B) $x + 3$
- C) $x - \frac{6}{(2x + 3)}$
- D) $x + 2 - \frac{6}{(2x + 3)}$

19

$$\frac{3x + 13}{(x + 3)^2} - \frac{3}{x + 3}$$

The expression above is equivalent to $\frac{b}{(x + 3)^2}$, where b is a positive constant and $x \neq -3$. What is the value of b ?

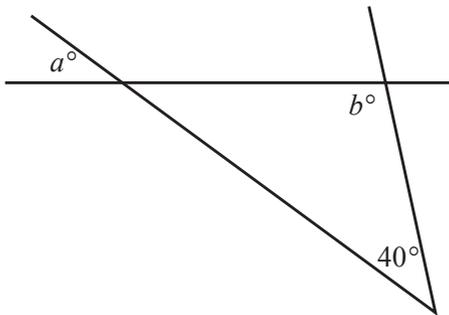
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Versatility of Making Up Numbers

Picking your own numbers works well on algebra problems, and is also effective on other types of problems: geometry, arithmetic, functions, etc.

In geometry problems with variables, making up numbers is a great strategy. However, you must obey the rules of geometry and the conditions stated in the problem.

20 



In the figure above, the value of a is equivalent to which of the following expressions?

- A) $40^\circ - b$
- B) $40^\circ + b$
- C) $180^\circ - b$
- D) $140^\circ - b$

13 

At a bakery, a baker can make m batches of cookies by adding n bags of cookie mix to water. If $n = m + 3$, how many additional bags of cookie mix are needed to make each additional batch of cookies?

- A) 0
- B) 1
- C) 2
- D) 3



Functions Drill 4

9

$$\sqrt{x+7} = x-5$$

What is the solution set of the equation above?

- A) {2}
- B) {4}
- C) {9}
- D) {2, 9}

15

If $(ax+3)(bx+4) = 18x^2 + 30x + 12$ for all values of x , what are the two possible values for a ?

- A) 3 and 4.5
- B) 4 and 6
- C) 3 and 4
- D) 2 and 9

24

$$h(t) = -10t^2 + 85t + 45$$

The equation above expresses the height h , in feet, of an object above ground t seconds after being thrown through the air. What does the number 45 represent in the equation?

- A) The maximum height, in feet, of the object
- B) The initial height, in feet, of the object
- C) The maximum speed, in feet per second, of the object
- D) The initial speed, in feet per second, of the object

25

$$h(t) = -4.9t^2 + 19.6t$$

The equation above expresses the height h , in meters, of a projectile t seconds after it is launched vertically upward from the ground with an initial velocity of 19.6 meters per second. How long will it take for the projectile to get back to the ground?

- A) 3.5
- B) 4.0
- C) 4.5
- D) 5.0

28

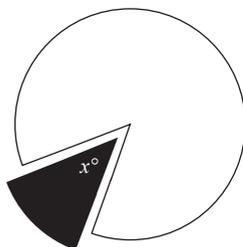
Given the functions $f(x)$ and $g(x)$ below, which of the following forms the composite function

$$g(f(x)) = \sqrt{x^2 + 2x}?$$

- | | $g(x)$ | $f(x)$ |
|----|----------------|---------------|
| A) | x | $\sqrt{x+2x}$ |
| B) | $2x^2$ | $\sqrt{x+2}$ |
| C) | $\sqrt{x^2+x}$ | x |
| D) | \sqrt{x} | x^2+2x |

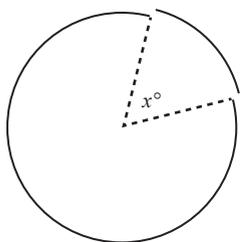


Area of Sector and Arc Length



Sector area is a fraction of the area of the circle

$$\frac{\text{Sector Area}}{\text{Circle Area}} = \frac{\text{Central Angle}}{360^\circ}$$

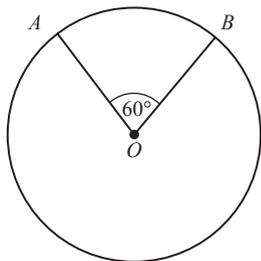


Arc length is a fraction of the circumference

$$\frac{\text{Arc Length}}{\text{Circumference}} = \frac{\text{Central Angle}}{360^\circ}$$

Both the area of a sector of a circle and its arc length are dependent on the central angle. The ratio of the area of a sector (arc length) to the area of the circle (circumference of the circle) will be equivalent to the ratio of the central angle to 360° .

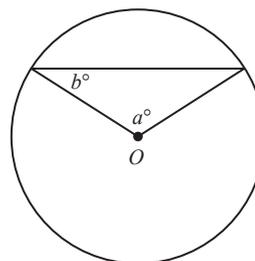
27



In the figure above, point O is the center of the circle. If the radius is 6, then what is area of sector AOB ?

- A) 6π
- B) 9π
- C) 18π
- D) 36π

28



In the figure above, point O is the center of the circle. If $b = 30^\circ$ and the radius is 6, then what is the length of the arc associated with the central angle a ?

- A) 2π
- B) 3π
- C) 4π
- D) 6π