



The ACT

TUTORING
MACHINES

ACT Manual

by Kimani Williams, PhD

© 2016 by Kimani Williams, PhD

ALL RIGHTS RESERVED. This book contains material protected under International and Federal Copyright Laws and Treaties. Any unauthorized reprint or use of this material is prohibited. No part of this book may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or by any information storage and retrieval system without express written permission from Kimani Williams, PhD.

Note From The Author

Kimani “The Machine” Williams, PhD

Ever since I have known myself, I have been tutoring my friends and fellow students in academia. I have been blessed with a knack for simplifying what seems complicated to students. After high school, I left my home country of Jamaica to go to Macalester College, a small liberal arts college in Minnesota.

After four wonderful years of Minnesota in the frigid tundra, it was time to return to a climate more favorable to a Jamaican, so I decided to further my studies by joining the PhD program in Electrical Engineering at the University of California, Santa Barbara. While working on my doctorate, I started tutoring part-time for a large test prep company. It so happens, that I am phenomenal at dramatically improving students’ test scores. Soon, I was tutoring so many students that I began to feel more like a full-time tutor and less like a PhD student.

After completing my PhD in Electrical Engineering, I decided to fully devote myself to my passion for teaching and tutoring, which led me to start my own company, Tutoring Machines, LLC. In my own microcosmic way, I am changing the world, one student at a time as I help students reach their goals through higher education.

Contents

General Introduction	10	1.5 Rhetorical Skills	83
ACT Structure	12	Rhetorical Skills	84
1.0 ACT English	17	Drill 1: Rhetorical Skills	90
ACT English Test	18	Drill 2: Putting it All Together.....	94
Scoring The English Test	19	Rhetorical Skills Overview	97
1.1 Grammar and Usage	20	1.6 ACT Practice English Test	99
Grammar and Usage	21	2.0 ACT Math	116
General Approach to English Passages	38	2.1 ACT Math Introduction	118
English: Drill	39	Content of the ACT Math Test	119
Grammar and Usage Overview.....	42	Common ACT Math Formulas	120
1.2 Grammar and Usage Homework	43	Structure of the ACT Math Test	122
Drill 1: Verbs	44	Developing Good Habits	123
Drill 2: Pronouns	48	Scoring the Math Test	124
Drill 3: Idioms and More.....	51	2.2 The Numbers Game	125
Drill 4: Passage V.....	53	Making Up Numbers	126
1.3 Punctuation and Sentence Structure ..56		“Must be” Questions.....	128
Punctuation and Sentence Structure	57	Versatility of Making Up Numbers.....	129
Drill 1: Passage I.....	69	Using the Answer Choices.....	131
Punctuation and Sentence Structure Overview	72	Systems of Equations.....	133
1.4 Punctuation and Sentence Structure Homework	73	Making Up Numbers Overview.....	134
General Approach to Punctuation and Sentence Structure	74	2.3 The Numbers Game Homework	135
Drill 1.....	74	Making Up Numbers Drill 1	136
Drill 2.....	77	Making Up Numbers Drill 2	137
Drill 3: Passage II.....	80	Making Up Numbers Drill 3	139
		Making Up Numbers Drill 4	140
		Making Up Numbers Drill 5	141
		Making Up Numbers Drill 6	142

2.4 Fundamentals	143	2.9 Arithmetic Homework	199
Fundamentals	144	Arithmetic Drill 1	200
Fundamentals Overview	156	Arithmetic Drill 2	201
2.5 Fundamentals Homework	157	Arithmetic Drill 3	202
Fundamentals Drill 1	158	Arithmetic Drill 4	203
Fundamentals Drill 2	159	Arithmetic Drill 5	204
Fundamentals Drill 3	160	2.10 Data Analysis	205
Fundamentals Drill 4	161	Mean, Mode, and Median	206
Fundamentals Drill 5	162	Arrangements	208
2.6 Geometry	163	Probability	209
General Approach to Geometry Problems ...	164	Factorial	211
Angles	165	Combinations and Permutations	211
Triangles	168	Sequences	213
Circles	171	Logic	214
Polygons and 3D	174	Matrices	215
Coordinate Geometry	175	Complex Numbers	216
Geometry Overview	176	Data Analysis Overview	217
2.7 Geometry Homework	175	2.11 Data Analysis Homework	218
Geometry Drill 1: Angles	178	Data Analysis Drill 1: Mean/Mode/Median ...	219
Geometry Drill 2: Triangles	180	Data Analysis Drill 2: Arrangements	221
Geometry Drill 3: Circles	183	Data Analysis Drill 3:	
Geometry Drill 4: Distance/Slope/Midpoint..	185	Probability/Factorial/Combinations and	
Geometry Drill 5: Polygons/3D	186	Permutations	222
2.8 Arithmetic	188	Data Analysis Drill 4: Sequences/Logic	224
Math Translation	189	Data Analysis Drill 5: Question Sets	225
Percentages	190	Data Analysis Drill 6:	
Ratio	192	Matrices/Complex Numbers	226
Variation	194	2.12 Functions	227
Quadratic Equations	196	Linear Equations	228
Arithmetic Overview	198	Functions	230
		Quadratic Functions	235
		Graph Translation	236
		Logarithms	238

Functions Overview.....	239	3.4 ACT Practice Reading Test.....	317
2.13 Functions Homework.....	240	4.0 ACT Science	330
Functions Drill 1.....	241	Structure of the ACT Science Test.....	332
Functions Drill 2.....	243	Section 4 - Science	332
Functions Drill 3.....	244	Scoring the Science Test.....	336
Functions Drill 4.....	245	4.1 Data Representation and Research	
Functions Drill 5.....	247	Summaries	335
2.14 ACT Practice Math Test.....	249	Order Matters.....	336
3.0 ACT Reading	274	General Approach to Science Passages.....	337
Structure of the ACT Reading Test.....	275	4.2 Conflicting Viewpoints.....	350
Scoring the Reading Test.....	276	Conflicting Viewpoints.....	351
3.1 Strategy for Reading Passages.....	279	General Approach to Conflicting	
Question Types: Specific Questions	280	Viewpoints.....	353
Question Types: General Questions	282	4.3 ACT Practice Science Test.....	365
Specific or General?	284	5.0 ACT Writing (Essay)	378
General Approach to Reading Passages.....	285	Tackling the ACT Essay.....	380
Reading Strategy Overview.....	291	Key Components of Top-Scoring Essays.....	381
3.2 Strategy for Reading Passages		The Template of the ACT Essay	383
Homework.....	292	The Introduction	387
General Approach to Reading Passages.....	293	The Body Paragraphs	388
Drill 1.....	294	The Conclusion	391
Drill 2.....	296	Essay Overview	392
Drill 3	298	5.1 ACT Writing (Essay) Homework.....	393
Drill 4	300	6.0 Homework Answers	410
3.3 Order Matters.....	302	English Homework	412
Order Matters.....	303	ACT English Practice Test	413
Comparative Passages	309	Math Homework	415
Literary Devices	313	ACT Math Practice Test	419
Order Matters Overview.....	316		

Reading Passages Homework 421

ACT Reading Practice Test 421

ACT Science Practice Test 423



Comparative and Superlative Modifiers

The comparative is used to compare two things. The superlative is used to compare three or more things.

➤ **Comparative:** *adjective + “er” / more + adjective*

*Example: Right: Dwight is **taller** than James. However, James is much **more athletic** than Dwight.*

*Wrong: Dwight is **more taller** than James. However, James is much **athleticer** than Dwight.*

➤ **Superlative:** *adjective + “est” / most + adjective*

*Example: Right: Dwight is the **tallest** player on the basketball team. He is the **most dominant** center in the league.*

*Wrong: Dwight is the **most tallest** player on the basketball team. He is the **dominantest** center in the league.*

Idioms

Idioms are the rules that are unique to the English language. The idioms tested on the ACT are primarily tested on phrases that require a specific preposition to be correct.

A preposition is a word that shows the relationship between a noun or pronoun and other words in the sentence (e.g. between, over, on, by, of, from, to).



Practice: Circle the Correct Idiom

1. Mary’s effort was much more (than / of / to) what was expected.
2. As the leader, he felt the need to assert his authority (from / over / of / as) the others.
3. The dog caused harm to the plant (on / by) accidentally stepping on the roots.



English: Drill

(9 minutes or 13.5 minutes with extended time)

PASSAGE III

Doctors Without Borders

Doctors Without Borders are a group of physicians
₃₁

- 31. A. NO CHANGE
- B. is
- C. was
- D. were

that gives emergency aid to people affected over wars,
₃₂ epidemics, natural and manmade disasters. The group was created in 1971 with 300 volunteers, doctors, nurses, and other staff, including the 13 founding doctors and journalists.

- 32. F. NO CHANGE
- G. on
- H. by
- J. about

While on its missions, the group witnessed many medical
₃₃ situations that affected underprivileged citizens of

- 33. A. NO CHANGE
- B. it's
- C. their
- D. there

the world but notice was not largely given to this because
₃₄ many people remained uninterested. Therefore, journalists in the organization garnered support for their cause by promoting modern advocacy, implementing media strategies, and utilize social networks.
₃₅

- 34. F. NO CHANGE
 - G. they went largely unnoticed
 - H. this was not a largely noticeable situation
 - J. these situations went largely unnoticed
- 35. A. NO CHANGE
 - B. utilized social networks
 - C. utilizing social networks
 - D. in the utilizing of social networks

Therefore, an **incomplete idea (sentence fragment)** is simply an idea that is not complete. A sentence fragment is often called a dependent clause since a dependent clause has a subject and a verb but an unfinished idea. A dependent clause needs to be paired with a complete idea to become complete.

1. Because you loved it. (**complete / incomplete**)
2. I bought you the necklace because you loved it. (**complete / incomplete**)
3. Since you paid for dinner. (**complete / incomplete**)
4. Since you paid for dinner, I will pay for dessert. (**complete / incomplete**)

The 2 Cs – Complete and Complete

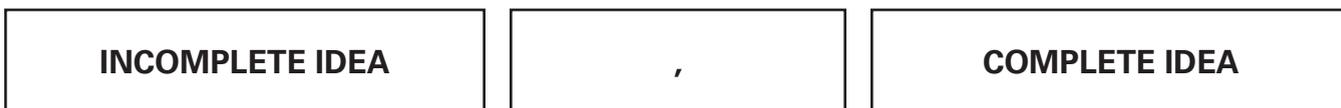
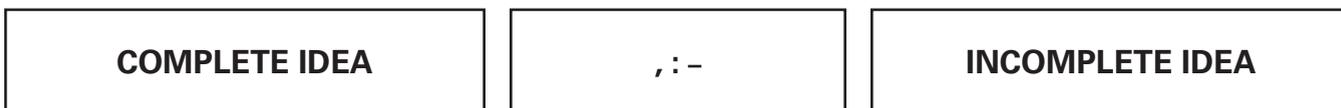
Two complete ideas can **ONLY** be separated by a period (.), semicolon (;), exclamation mark (!), question mark (?), single dash (-), colon (:), and a comma + FANBOYS (for, and, nor, but, or, yet, so).



Anything Other Than the 2 Cs – Complete and Incomplete

A comma **CANNOT** be used to separate two complete ideas. A comma separating two complete ideas is known as a comma splice, which is grammatically incorrect. Two complete ideas joined without any punctuation is called a run-on sentence, which is also grammatically incorrect.

A comma can be used to separate a complete and incomplete idea. A colon or a single dash can also be used to separate a complete and incomplete idea, but the colon or single dash has to follow the complete idea and come before the incomplete idea.

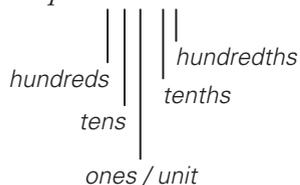




DIGIT

Any numeral from 0 to 9 that is a part of another number.

Example: 572.48



11. For how many integers between 33 and 44 is the ones digit greater than the tens digit?
- A. 4
B. 5
C. 6
D. 10
E. 11
50. The fraction $\frac{1}{7}$ is equivalent to $0.\overline{142857}$. What is the digit in the 501st decimal place of $0.\overline{142857}$?
- (Note: The digit in the 1st decimal place of $0.\overline{142857}$ is 1.)
- F. 1
G. 2
H. 4
J. 5
K. 8



CROSS MULTIPLICATION

Solve an equation with fractions on both sides of the equation by multiplying across the equal sign.

$$\frac{a}{b} = \frac{c}{d} \quad ad = bc$$

34. If $\frac{10x + 4y}{3y} = \frac{34}{3}$, then what is the value of $\frac{x}{y}$?
- F. $\frac{1}{3}$
G. 3
H. 4
J. 6
K. 13



Fundamentals Drill 5

8. The product of three consecutive prime numbers is 385. What is the smallest of the three numbers?
- F. 5
G. 7
H. 9
J. 11
K. 13
11. How many distinct positive integers are prime factors of both 40 and 48?
- A. 1
B. 2
C. 3
D. 4
E. 5
18. If $\frac{a+b}{3a+b} = \frac{1}{2}$ and a and b are positive integers, what is the least possible value of $a + b$?
- F. 2
G. 3
H. 4
J. 6
K. 8
31. Which of the following expressions is equal to $\frac{2}{4+\sqrt{3}}$?
- A. $\frac{2}{13}$
B. $\frac{2}{7}$
C. $\frac{8-2\sqrt{3}}{13}$
D. $\frac{8+2\sqrt{3}}{19}$
E. $\frac{8-\sqrt{6}}{13}$
58. The fraction $\frac{3}{7}$ is equivalent to $0.\overline{428571}$. What is the digit in the 272nd decimal place of $0.\overline{428571}$? (Note: The digit in the 1st decimal place of $0.\overline{428571}$ is 4.)
- F. 1
G. 2
H. 4
J. 5
K. 8
59. The solution set of which of the following equations is the set of numbers that are 10 units from -2 ?
- A. $|x - 2| = 10$
B. $|x + 2| = 10$
C. $|x - 10| = 2$
D. $|x + 10| = 2$
E. $|x + 10| = -2$

**MATH TEST***60 Minutes–60 Questions*

DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose,

but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates arithmetic mean.

1. The table below shows the time, in minutes, for 8 runners in a 20-mile race after each runner completed the race. What is the mean time of the 8 runners, to the nearest minute?

DO YOUR FIGURING HERE.

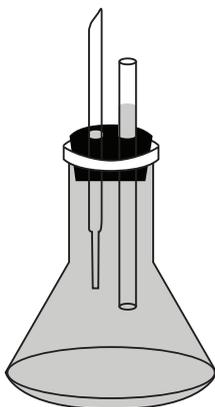
Runner	Time (minutes)
1	124
2	133
3	129
4	143
5	168
6	162
7	145
8	158
Sum	1,162

- A. 138
- B. 145
- C. 147
- D. 150
- E. 163

GO ON TO THE NEXT PAGE.

Passage IV

The experiments below were conducted to study the effects of temperature on the volume of a liquid. Students filled a number of conical flasks to the top with either distilled water or salt (CaCl) water at 25°C. Each flask was sealed with a rubber stopper containing a thermometer and a graduated glass tubing. The initial height of the liquid in the glass tubing was measured at 20 mm above the top of the stopper. The apparatus is shown in the figure below.



Experiment 1

The conical flask filled with distilled water was partially immersed in a cold bath at -4°C . As the temperature of the distilled water in the flask decreased, the height of the water in the glass tubing also decreased. At 4°C , the height of the water in the glass tubing stopped decreasing and began to increase. At 0°C , the distilled water began to form ice. At that point the students concluded the experiment.

Experiment 2

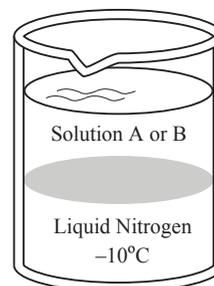
The conical flask filled with distilled water was slowly heated by a bunsen burner. As the temperature of the water increased, the height of the water in the glass tubing also increased. At 100°C , the distilled water began turning to steam. At that point the students concluded the experiment.

Experiment 3

Two conical flasks containing Solutions A and B were prepared with different salt concentrations. Solution A had a higher salt concentration than Solution B. Both solutions were cooled as in Experiment 1. In both flasks, the height of the salt water in the glass tubing decreased until the temperature reached -4°C but no freezing occurred. Freezing occurred for Solution B but not for Solution A at -8°C .

In addition, Solutions A and B were slowly heated. In both flasks, the height of the salt water increased and did not turn to steam at 100°C . However, the salt water bubbled out the top of the tubing at 103°C for Solution B and at 105°C for Solution A.

18. A hypothetical 2-partitioned beaker contains either Solution A or Solution B in the top partition and liquid nitrogen in the bottom partition as shown in the figure below.



If the barrier separating the 2 partitions could be removed without disturbing the liquid nitrogen or the solution, which of the following is most likely to occur?

- F. Solution A would begin to freeze and crack the glass of the beaker.
- G. Solution A would decrease in temperature until it got to -4°C and then no further temperature decrease would occur.
- H. Solution B would decrease in temperature but no freezing would occur.
- J. Solution B would decrease in temperature but eventually would start to freeze and crack the glass of the beaker.

Passage III

Introduction

Three students were given the following information on the conservation of energy:

- *Kinetic energy* is the energy that results from an object in motion, and *gravitational potential energy* is the energy that an object possesses based on its height above the ground.
- The *total mechanical energy* of an object is the sum of its kinetic energy and its gravitational potential energy. Energy is conserved when the total mechanical energy remains constant.
- Some energy will be lost in the presence of a frictional force, which would result in the total mechanical energy not being conserved.

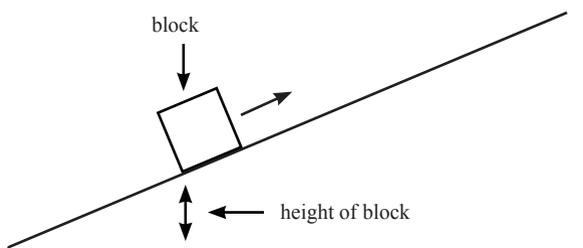
The students' teacher then described the following experiment:

Suppose a block was placed on a surface and was subjected to a force to cause motion. As the block moved along the surface, a student measured the speed of the block twice. The second measured speed was lower than the first.

Below are the viewpoints of three students about the 2 measurements of speed and their predictions about the motion of the block after the 2 measurements.

Student 1

At the time when the 2 measurements were made, the block was moving up a frictionless, inclined surface, and was slowing down at a constant rate (see figure below). No air was present.



The block would come to rest due to friction, and then accelerate down the incline due to gravity. At any given height, the block's speed moving up the incline would have been the same as its speed going down the incline.

Student 2

The block was moving on a horizontal surface. The block experienced a constant frictional force as it moved along the surface. This force alone resulted in the block slowing down at a constant rate and would have caused the block eventually to come to rest. Once stopped, the block would have remained at rest.

Student 3

The block moved on a horizontal surface that had no friction. However, the block experienced a force from air resistance. Air resistance alone slowed down the block and caused it to lose mechanical energy. The rate at which the block slowed down depended on the magnitude of the air resistance force. As the block's speed decreased, the amount of air resistance decreased.

15. Assuming that Student 2's viewpoint is correct, was the total mechanical energy of the block conserved while the block moved?
- A. Yes, because both the block's kinetic energy and its gravitational potential energy increased.
 - B. Yes, because both the block's kinetic energy and its gravitational potential energy decreased.
 - C. No, because the block's kinetic energy decreased and its gravitational potential energy remained constant.
 - D. No, because the block's kinetic energy increased and its gravitational potential energy decreased.